

Entanglement generation in relativistic cavity motion

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Abstract. We analyse particle creation and mode mixing for a quantum field in an accelerated cavity, assuming small accelerations but allowing arbitrary velocities, travel times and travel distances, and in particular including the regime of relativistic velocities. As an application, we identify a desktop experimental scenario where the mode mixing resonance frequency in linear sinusoidal motion or in uniform circular motion is significantly below the particle creation resonance frequencies of the Dynamical Casimir Effect, and arguably at the threshold of current technology. The mode mixing acts as a beamsplitter quantum gate, experimentally detectable not only via fluxes or particle numbers but also via entanglement generation.

1. Introduction and overview

As anyone caught by a police speed camera can testify, spacetime kinematics affects photons that are present in spacetime. For a relativistic quantum field, spacetime kinematics can also create particles where initially there were none, and the very notion of a “particle” depends on the motion of an observer. In flat spacetime, celebrated examples are the thermality seen in Minkowski vacuum by uniformly accelerated observers, known as the Unruh effect [1, 2], and the creation of particles by moving boundaries, known as the Dynamical (or non-stationary) Casimir Effect (DCE) [3, 4]. In curved spacetime, a celebrated example is the Hawking radiation emitted by black holes [5].

There has been significant recent interest in harnessing consequences of spacetime kinematics to serve quantum information tasks. The broad issue is how entanglement and the fidelity of quantum communication are affected by observer motion and by spacetime curvature [6, 7, 8, 9, 10, 11, 12, 13]. A specific question is how entanglement may be created and used to construct quantum gates [14, 15, 16, 17]. A recent survey can be found in [18].

In this contribution we consider a quantum field in an accelerated, perfectly confining cavity in Minkowski spacetime. As the cavity walls shield the field from acceleration horizons, the cavity field cannot be expected to exhibit thermality via the Unruh effect when the acceleration is linear and uniform [19]. While the absence of the conventional Unruh effect can hence be regarded as a disadvantage of a cavity field, the field will nevertheless respond to the cavity’s acceleration in

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a nontrivial fashion, and the confinement has technical advantages in that a discrete spectrum makes a number of entanglement quantifiers mathematically well defined [20] and conceptual advantages related to localisation issues in relativistic quantum measurement theory [21, 22]. Finally, a practical advantage of a cavity field is that the cavity can be given a variety of travel scenarios of interest, including periodic shakes that appear in the DCE literature [3, 4].

In Sections 2 and 3 we review a recently-developed formalism [23] for a relativistic field in a moving cavity that is assumed mechanically rigid, in the sense that the cavity's shape in its instantaneous rest frame remains unchanged even though the velocity may change in time. The acceleration is assumed to be small compared with c^2 divided by the cavity's size but may otherwise have arbitrary time dependence, both in magnitude and in direction. The velocities, travel times and travel distances are unrestricted, so that the treatment remains valid even when the velocities become relativistic. To linear order in the acceleration, the Bogoliubov coefficients between inertial motions at early and late times can then be expressed in terms of the Fourier transform of the cavity's acceleration.

In Section 4 we apply the formalism to two periodic motions, a linear sinusoidal motion and a uniform circular motion [23]. We identify a configuration that brings the mode mixing resonance frequency in these motions into an apparently experimentally feasible regime, significantly below the experimentally prohibitive particle creation resonance frequencies known from the DCE literature [3, 4]. A geometrically optimised desktop experimental scenario at optical wavelengths is seen to produce significant mode mixing within a millisecond of shaking at megahertz frequencies. From the quantum information viewpoint the mode mixing acts as a beamsplitter quantum gate. The experimental verification opportunities can hence utilise entanglement phenomena, in addition to conventional observations of fluxes or particle numbers.

We conclude in Section 5 by summarising the results and discussing the prospects to bridge the gap to an actual experiment that would create entanglement by mechanical motion on the desktop.

We set from now on $c = \hbar = 1$, reinstating units by dimensional analysis where appropriate.

2. Rigid cavity at small accelerations: (1+1) Minkowski

2.1. Setup

We start with a cavity in $(1 + 1)$ -dimensional Minkowski spacetime. The cavity is assumed mechanically rigid, in the sense that it maintains constant length L in its instantaneous rest frame. We denote by τ the proper time at the centre of the cavity and by $a(\tau)$ the proper acceleration at the centre of the cavity. To maintain rigidity, the acceleration must be bounded by $|a|L < 2$ [11].

The cavity contains a real scalar field ϕ of mass $\mu_0 \geq 0$, with Dirichlet boundary conditions at the walls.

We assume a to vanish at early and late times but to be nonvanishing at some intermediate times. An example where a is piecewise constant with exactly two points of discontinuity is shown in Figure 1.

The inertial segments of the cavity worldtube at early and late times are static: the cavity is dragged along a Minkowski time translation Killing vector. These timelike Killing vectors provide a distinguished definition of positive and negative frequencies tailored to the cavity's motion at early and late times, resulting to field modes with the angular frequencies $\omega_n = \sqrt{\mu_0^2 + (\pi n/L)^2}$, $n = 1, 2, \dots$, and quantisation leads to the familiar Fock space of an inertial cavity. However, because a is nonvanishing at intermediate times, the early time and late time vacua do not need to coincide, nor do the early time and late time notions of particles. The task is to find the Bogoliubov transformation that relates the early time and late time Fock spaces.

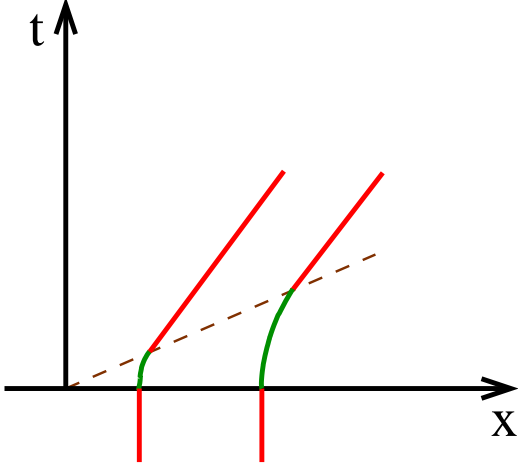


Figure 1. A $(1+1)$ cavity worldtube in which the initial and final inertial segments are joined by one segment of uniform acceleration. During the accelerated segment the cavity is static with respect to the boost Killing vector $x\partial_t + t\partial_x$, whereas during each of the inertial segments the cavity is static with respect to a Minkowski time translation Killing vector.

2.2. Solution

When the accelerations are so small that $|a|L \ll 1$, the problem has a solution that builds on the following three key observations [11, 23].

First, while a cavity with constant nonvanishing a is not inertial, it is nevertheless static: the cavity is dragged along a boost Killing vector, as shown in Figure 1. The cavity can be regarded as uniformly accelerated, even though the value of the acceleration varies between spatial points within the cavity, attaining the values $2a/(2 \pm aL)$ at the two walls and being equal to a only at the centre. The boost Killing vector is timelike and provides now a distinguished definition of positive and negative frequencies tailored to the cavity's motion. For $\mu_0 > 0$, the field modes can be written down in terms of modified Bessel functions and the angular frequencies are determined implicitly by a transcendental equation. For $\mu_0 = 0$, the field modes and their angular frequencies have elementary expressions [11].

Second, the Bogoliubov transformation between an inertial segment and a uniformly accelerated segment has a Maclaurin expansion in the dimensionless parameter aL , involving at each order only elementary expressions, and for $|a|L \ll 1$ the linear order terms provide already a good approximation [11]. Cavity world tubes for which a is piecewise constant with a finite number of discontinuities can hence be treated by composing this inertial-to-uniform transformation, its inverse, and the free evolution of the field modes during the uniformly-accelerated segments [11, 15]. The prototype cavity worldtube in which the initial and final inertial segments are joined by a single segment of uniform acceleration is shown in Figure 1.

Third, acceleration that is not necessarily piecewise constant can be handled by passing to the limit [23]. The Bogoliubov coefficient matrices α and β from the initial inertial segment to the final inertial segment have the expansions

$$\alpha = e^{i\omega(\tau_1 - \tau_0)}(1 + \hat{A} + O(h^2)), \quad (1)$$

$$\beta = e^{i\omega(\tau_1 - \tau_0)}\hat{B} + O(h^2), \quad (2)$$

where $h(\tau) := La(\tau)$, $\omega = \text{diag}(\omega_1, \omega_2, \dots)$,

$$\hat{A}_{nn} = 0, \quad (3)$$

$$\hat{A}_{mn} = -i \frac{\pi^2 mn(1 - (-1)^{m+n})}{L^4 (\omega_m - \omega_n)^2 \sqrt{\omega_m \omega_n}} \int_{\tau_0}^{\tau_1} e^{-i(\omega_m - \omega_n)(\tau - \tau_0)} h(\tau) d\tau \quad \text{for } m \neq n, \quad (4)$$

$$\hat{B}_{mn} = i \frac{\pi^2 mn(1 - (-1)^{m+n})}{L^4 (\omega_m + \omega_n)^2 \sqrt{\omega_m \omega_n}} \int_{\tau_0}^{\tau_1} e^{-i(\omega_m + \omega_n)(\tau - \tau_0)} h(\tau) d\tau, \quad (5)$$

the initial inertial segment ends at $\tau = \tau_0$ and the final inertial segment begins at $\tau = \tau_1$. To linear order in h , the Bogoliubov coefficients are hence obtained by just Fourier transforming the acceleration.

2.3. Outcomes

While the perturbative solution (1)–(5) for the Bogoliubov coefficients assumes the acceleration to be so small that $|h| \ll 1$ over the cavity’s worldtube, the velocities, travel times and travel distances remain unrestricted. The solution remains in particular valid even when the velocities grow relativistic. In this sense our perturbative treatment is complementary to the small distance approximations often considered in the DCE literature [3, 4], while of course overlapping in the common domain of validity.

\hat{A}_{mn} and \hat{B}_{mn} scale linearly in h , but their magnitudes depend also crucially on whether h changes slowly or rapidly compared with the oscillating integral kernels in (4) and (5). It is useful to consider the two extreme limits individually.

One extreme is the limit of slowly-varying h . In this limit \hat{A}_{mn} and \hat{B}_{mn} vanish, as is seen from (4) and (5) by the Riemann-Lebesgue lemma, and as can also be argued on more general adiabaticity grounds [19, 24]. The Bogoliubov coefficients indeed evolve by pure phases over any segment in which h is constant, not just to linear order in h as seen from (4) and (5) but also nonperturbatively in h [11, 23]. Particles in the cavity are hence created by *changes* in the acceleration, rather than by acceleration itself. The cavity is in this respect similar to a single accelerating mirror, which radiates only when its acceleration changes in time [25].

The other extreme is the case of piecewise constant h . In this case the contributions to \hat{A}_{mn} and \hat{B}_{mn} come entirely from the discontinuous jumps in h . While this extreme may not be experimentally realisable by a material cavity, it can be simulated by a cavity whose walls are static dc-SQUIDS undergoing electric modulation that simulates mechanical motion [26, 27].

When h is sinusoidal and its angular frequency ω_r equals the angular frequency of the integral kernel in (4) or (5), the corresponding Bogoliubov coefficient will grow linearly in the duration of the sinusoidal motion. These resonance conditions read

$$\hat{A}_{mn} : \omega_r = |\omega_m - \omega_n| , \quad (6)$$

$$\hat{B}_{mn} : \omega_r = \omega_m + \omega_n , \quad (7)$$

where in each case $m - n$ needs to be odd in order for the coefficient to be nonvanishing. The particle creation resonance (7) is well known in the DCE literature [3, 4, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38]. The mode mixing resonance (6) is also known [29, 30, 31, 32, 33, 34, 35] but seems to have received significant attention only in situations where it happens to coincide with a particle creation resonance.

3. Rigid cavity at small accelerations: (3+1) Minkowski

The above $(1+1)$ -dimensional analysis generalises immediately to a rectangular cavity of edge lengths (L_x, L_y, L_z) in $(3+1)$ -dimensional Minkowski spacetime provided the acceleration keeps pointing in one of the cavity’s three principal directions. The quantum numbers in the transverse directions remain inert, just contributing to the effective $(1+1)$ -dimensional mass μ_0 . Note that μ_0 is hence positive even when the $(3+1)$ -dimensional field is massless.

It is further immediate to compose segments in which the acceleration points in one of the cavity’s principal directions, as seen in the cavity’s instantaneous rest frame, even if this direction differs from segment to segment.

Acceleration of unrestricted magnitude and direction would require new input regarding how the shape of the cavity responds to such acceleration. To *linear* order in the acceleration, however, boosts in different spatial directions commute, and we can treat acceleration as a vector

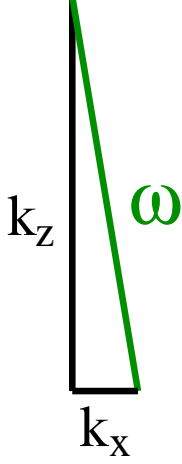


Figure 2. A momentum space configuration in which the mode mixing resonance angular frequency (8) is significantly lower than the angular frequencies of the individual cavity modes. The momenta of the quanta are aligned in the z -direction to a high degree of accuracy, so that $k_z \gg (k_x^2 + k_y^2)^{1/2}$. When k_x and k_y change by the respective small amounts Δk_x and Δk_y , the changes in the angular frequency ω are much smaller than Δk_x and Δk_y .

superposition of accelerations in the three principal directions in the cavity's instantaneous rest frame [23]. For sinusoidal acceleration with angular frequency ω_r , the resonance conditions (6) and (7) generalise to

$$\hat{A}_{mnp, m'n'p'} : \omega_r = |\omega_{mnp} - \omega_{m'n'p'}|, \quad (8)$$

$$\hat{B}_{mnp, m'n'p'} : \omega_r = \omega_{mnp} + \omega_{m'n'p'}, \quad (9)$$

where $\omega_{mnp} = \sqrt{\mu^2 + (\pi m/L_x)^2 + (\pi n/L_y)^2 + (\pi p/L_z)^2}$, μ is the mass of the field, the quantum numbers (m, n, p) are positive integers, and the difference in the quantum number in the direction of oscillation needs to be odd.

4. Desktop mode mixing experiment with periodic motion

The particle creation resonance angular frequency (9) is comparable in magnitude to the angular frequencies of the individual cavity modes and appears prohibitively difficult to realise in experiments with mechanical oscillations [3, 4]. The mode mixing resonance angular frequency (8) can however be significantly lower when the cavity modes are highly transverse to the direction of the oscillation, as illustrated in Figure 2. We now outline a scenario that optimises this lowering [23].

4.1. Scenario

We consider a massless field in $(3+1)$ dimensions and focus on quanta whose wavelength λ is much smaller than any of the cavity's edge lengths. The crucial input is to align the momenta close to the z -direction, as shown in Figure 2, so that $\omega_{mnp} \approx 2\pi/\lambda + \frac{1}{4}\pi\lambda[(m/L_x)^2 + (n/L_y)^2]$. For small changes in m and n , the changes in ω_{mnp} are then much smaller than the changes in the horizontal wave numbers $\pi m/L_x$ and $\pi n/L_y$.

We let the cavity undergo linear or circular harmonic oscillation orthogonal to the z -direction, with amplitude r_x (r_y) in the x -direction (y -direction). For motion in x -direction, the mode mixing resonance angular frequency (8) between modes m and m' , with $m - m'$ odd, is

$$\omega_r \approx \frac{1}{4}\pi\lambda L_x^{-2} |m^2 - (m')^2|, \quad (10)$$

and the growth rate of the mode mixing Bogoliubov coefficient is

$$\frac{d}{d\tau} |\hat{A}_{\text{res}}| \approx \frac{1}{2}\pi m m' r_x \lambda L_x^{-3}. \quad (11)$$

The lowest resonance occurs for $m = 1$ and $m' = 2$. Similar formulas ensue for the y -resonance, and for circular motion both resonances are present.

The experimental scenario starts by first trapping one or more quanta in the cavity, in modes whose momenta are aligned close to the z -direction. The cavity is then made to oscillate perpendicularly to the z -direction, linearly or circularly. Finally, a measurement on the quantum state of the cavity is performed. The resonance mode mixing that takes place during the acceleration is assumed to dominate any effects due to the initial trapping and the final releasing of the quanta.

4.2. Desktop numbers

For concrete numbers, we take $\lambda = 600 \text{ nm}$ and $L_x = L_y = 1 \text{ cm}$ (the value of L_z does not need to be specified provided $L_z \gg \lambda$). The lowest resonance angular frequency is $\omega_r \approx 4.2 \times 10^6 \text{ s}^{-1}$, corresponding to an oscillation frequency 0.7 MHz . With amplitude $1 \mu\text{m}$, (11) and its y -counterpart show that the mode mixing Bogoliubov coefficient grows to order unity within a millisecond. Following the growth further would require theory beyond our perturbative treatment.

For linear motion, oscillation of micron amplitude at megahertz frequency may be achievable by using ultrasound to accelerate the cavity. Storing the quantum in the cavity for a millisecond could be challenging although recent advances indicate that it may be feasible [39].

For circular motion, the threshold angular velocity $\omega_r \approx 4.2 \times 10^6 \text{ s}^{-1} \approx 4 \times 10^7 \text{ rpm}$ exceeds the angular velocity $1.5 \times 10^5 \text{ rpm}$ achieved by medical ultracentrifuges [40], although only by about two orders of magnitude, and it is conceivable that this gap could be bridged by a specifically designed system.

5. Conclusions and outlook

We have quantised a scalar field in a rectangular cavity that is accelerated arbitrarily in $(3+1)$ -dimensional Minkowski spacetime, in the limit of small accelerations but arbitrary velocities and travel times. The Bogoliubov coefficients between inertial motion at early and late times were expressed in terms of the Fourier transform of the acceleration. For linear or circular periodic motions, we identified a configuration in which the mode mixing resonance frequency is significantly below the frequencies of the cavity modes.

It can be verified that our scalar field analysis adapts in a straightforward way to a Maxwell field with perfect conductor boundary conditions: the two polarisation modes reduce to $(1+1)$ scalar fields with respectively Dirichlet and Neumann boundary conditions, and the Neumann condition Bogoliubov coefficients obey estimates that are qualitatively similar to those given above [41]. The mode mixing effects appear hence to be within the reach of a desktop experiment with photons, achievable with current technology in its mechanical aspects, if perhaps not yet in the storage capabilities required of a mechanically oscillating optical cavity.

Assuming the cavity to be rectangular allowed us to treat the cavity as spatially rigid in the small acceleration limit even when the velocities may be relativistic and the direction of the acceleration may vary in time. We anticipate that the particle creation and mode mixing effects are not qualitatively sensitive to the detailed shape of the cavity, and this freedom could be utilised in the development of a concrete laboratory implementation.

The experimental prospects could be further improved by filling the cavity with a medium that slows light down [42]. As the medium breaks Lorentz-invariance, our analysis would not be directly applicable at relativistic velocities, but at nonrelativistic velocities it may suffice to just include appropriate slowing-down numerical factors in the angular frequencies.

Our experimental scenario involves significant mode mixing but no significant particle creation. One might therefore be tempted to dismiss the scenario as a glorified speed camera effect and question the interest of a laboratory observation. However, mode mixing without

particle creation is known in quantum optics as a passive transformation [43], normally implemented by passive optical elements such as beam splitters and phase plates, and these transformations have a well understood capability to create and degrade entanglement [14, 16, 44, 45]. For example, the mode mixing generates entanglement from an initial Gaussian state only if this state is squeezed [16, 44, 45]. The oscillating cavity can hence be tuned to act as a beam splitter quantum gate, creating or degrading entanglement in situations where particles are initially present. This means that the mode mixing could be verified experimentally by observations of entanglement, and the entangling power of the system could be potentially harnessed to quantum information tasks. We anticipate that observations of entanglement will generally provide opportunities for experimental verification of both particle creation and mode mixing effects that are complementary to observations of fluxes or particle numbers.

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References

- [1] Unruh W G 1976 *Phys. Rev. D* **14** 870
- [2] Crispino L C B, Higuchi A and Matsas G E A 2008 *Rev. Mod. Phys.* **80** 787 (*Preprint* 0710.5373 [gr-qc])
- [3] Dodonov V V 2001 *Adv. Chem. Phys.* **119** 309 (*Preprint* quant-ph/0106081)
- [4] Dodonov V V 2010 *Phys. Scripta* **82** 038105
- [5] Hawking S W 1975 *Commun. Math. Phys.* **43** 199 [Erratum 1976 *Commun. Math. Phys.* **46** 206 (1976)]
- [6] Alsing P M and Milburn G J 2003 *Phys. Rev. Lett.* **91** 180404 (*Preprint* quant-ph/0302179)
- [7] Fuentes-Schuller I and Mann R B 2005 *Phys. Rev. Lett.* **95** 120404 (*Preprint* quant-ph/0410172)
- [8] Bruschi D E, Louko J, Martín-Martínez E, Dragan A and Fuentes I 2010 *Phys. Rev. A* **82** 042332 (*Preprint* 1007.4670 [quant-ph])
- [9] Downes T G, Fuentes I and Ralph T C 2011 *Phys. Rev. Lett.* **106** 210502 (*Preprint* 1007.4035 [quant-ph])
- [10] Bruschi D E, Dragan A, Fuentes I and Louko J 2012 *Phys. Rev. D* **86** 025026 (*Preprint* 1205.5296 [quant-ph])
- [11] Bruschi D E, Fuentes I and Louko J 2012 *Phys. Rev. D* **85** 061701 (*Preprint* 1105.1875 [quant-ph])
- [12] Friis N, Lee A R, Bruschi D E and Louko J 2012 *Phys. Rev. D* **85** 025012 (*Preprint* 1110.6756 [quant-ph])
- [13] Downes T G, Ralph T C and Walk N 2012 *Preprint* 1203.2716 [quant-ph]
- [14] Friis N, Bruschi D E, Louko J and Fuentes I 2012 *Phys. Rev. D* **85** 081701 (*Preprint* 1201.0549 [quant-ph])
- [15] Bruschi D E, Dragan A, Lee A R, Fuentes I and Louko J 2012 *Preprint* 1201.0663 [quant-ph]
- [16] Friis N and Fuentes I 2012 *J. Mod. Opt.* online (*Preprint* 1204.0617 [quant-ph])
- [17] Friis N, Huber M, Fuentes I and Bruschi D E 2012 *Phys. Rev. D* **86** 105003 (*Preprint* 1207.1827 [quant-ph])
- [18] Rideout D *et al* 2012 *Class. Quantum Grav.* **29** 224011 (*Preprint* 1206.4949 [quant-ph])
- [19] Schützhold R and Unruh W G 2005 *Preprint* quant-ph/0506028
- [20] Plenio M B and Virmani S 2007 *Quant. Inf. Comput.* **7** 1 (*Preprint* quant-ph/0504163)
- [21] Sorkin R D 1993 *Directions in General Relativity: Proceedings of the 1993 International Symposium, College Park, Maryland, 1993* vol 2 ed B L Hu and T A Jacobson (Cambridge, England: Cambridge University Press) p 293 (*Preprint* gr-qc/9302018)
- [22] Benincasa D M T, Borsten L, Buck M and Dowker F 2012 *Preprint* 1206.5205 [quant-ph]
- [23] Bruschi D E, Louko J, Faccio D and Fuentes I 2012 *Preprint* 1210.6772 [quant-ph]
- [24] Unruh W G 2012 private communication
- [25] Davies P C W and Fulling S A 1976 *Proc. Roy. Soc. Lond. A* **348** 393
- [26] Wilson C M, Johansson G, Pourkabirian A, Simoen M, Johansson J R, Duty T, Nori F and Delsing P 2011 *Nature* **479** 376 (*Preprint* 1105.4714 [quant-ph])
- [27] Friis N, Lee A R, Truong K, Sabín C, Solano E, Johansson G and Fuentes I 2012 *Preprint* 1211.5563 [quant-ph]
- [28] Lambrecht A, Jaekel M-T and Reynaud S 1996 *Phys. Rev. Lett.* **77** 615
- [29] Mundarain D F and Maia Neto P A 1998 *Phys. Rev. A* **57** 1379 (*Preprint* quant-ph/9808064)
- [30] Crocce M, Dalvit D A R and Mazzitelli F D 2001 *Phys. Rev. A* **64** 013808
- [31] Crocce M, Dalvit D A R and Mazzitelli F D 2002 *Phys. Rev. A* **66** 033811 (*Preprint* quant-ph/0205104)
- [32] Ruser M 2006 *Phys. Rev. A* **73** 043811 (*Preprint* quant-ph/0509030)

- [33] Yuce C and Ozcakmakli Z 2008 *J. Phys. A* **41** 265401 (*Preprint* 0802.3765 [hep-th])
- [34] Dodonov A V and Dodonov V V 2001 *Phys. Lett. A* **289** 291 (*Preprint* quant-ph/0109019)
- [35] Yuce C and Ozcakmakli Z 2009 *J. Phys. A* **42** 035403 (*Preprint* 0811.0665)
- [36] Schützhold R, Plunien G and Soff G 1998 *Phys. Rev. A* **57** 2311 (*Preprint* quant-ph/9709008)
- [37] Dodonov V V and Andreato M A 1999 *J. Phys. A* **32** 6711 (*Preprint* quant-ph/9908038)
- [38] Dodonov A V, Dodonov V V and Mizrahi S S 2005 *J. Phys. A* **38** 683
- [39] Kuhr S *et al* 2007 *Appl. Phys. Lett.* **90** 164101 (*Preprint* quant-ph/0612138)
- [40] Website <http://www.thermoscientific.com>, visited 29 September 2012
- [41] Louko J 2013 (in preparation)
- [42] Lauprêtre T, Proux C, Ghosh R, Schwartz S, Goldfarb F and Bretenaker F 2011 *Opt. Lett.* **36** 1551 (*Preprint* 1104.0158 [physics.optics])
- [43] Simon R, Mukunda N and Dutta B 1994 *Phys. Rev. A* **49** 1567
- [44] Wolf M M, Eisert J and Plenio M B 2003 *Phys. Rev. Lett.* **90** 047904 (*Preprint* quant-ph/0206171)
- [45] Alsing P M and Fuentes I 2012 *Class. Quantum Grav.* **29** 224001 (*Preprint* 1210.2223 [quant-ph])